

**Mathematics Standards**

**Pre-Calculus**

**Course Overview:** In this course, students will explore a variety of functions in multiple ways and develop concepts which are used in calculus. This course will include the study of trigonometry beginning with cyclic functions and continuing later with analytic trigonometry. Other topics include matrices, advanced algebraic techniques, area under a curve, and an introduction to limits. The students will also learn to write programs on their graphing calculator.

**Bold standards are essential standards that all students will learn as they complete the course.**

**Unit 1: Tools for the Journey (approximately 14 days)**

**Description:** This unit will provide students with background ideas that will be used later in the course. Many of the ideas are extensions of concepts from previous courses.

**Standards**

1. Students will sketch graphs as transformations from parent graphs. (F-BF.3)

2. Students will combine functions using a variety of operations including composition and they will find and use inverses of functions. (F-BF.1)

3. Students will find and use inverses of functions. (F-BF.4)

4. Students will use transformations on graphs of functions that are not parent graphs and functions that are not explicitly defined. (F-BF.3)

5. Students will be introduced to programming on the graphing calculator.

6. **Students will find the equation for point-slope form of a line and relate it to transformations of lines.** (F-IF.8)

7. Students will derive and apply the Law of Sines (G-SRT.9 (+), G-SRT.11 (+))

8. Students will derive and apply the SAS formula for the area of a triangle (G-SRT.9 (+))

9. Students will derive and apply the Law of Cosines (G-SRT.9 (+), G-SRT.11 (+))

**Unit 2: Trigonometric Functions (approximately 30 days)**

**Description:** In this unit, students will explore the relationship between right-triangle trigonometry and new cyclic functions. They will be introduced to a new representation that is useful for the study of trigonometric functions: a unit circle. They will also learn how to use radians instead of degrees to describe angles. They will transform trigonometric functions and find general equations for them. They will also learn about a new property that is characteristic of trigonometric functions called a period and they will write equations that describe a variety of periodic functions.

**Standards**

1. Students will use experimental data generated from measuring the heights of right triangles to create a sine graph. (F-IF.7e, F-TF.5)

2. Students will develop an understanding of reference angles and will explore the connections between the sine graph and the unit circle. (F-IF.7e)

3. Students will use the cosine function to calculate horizontal distances in a unit circle and they will draw conclusions about the relationships between the sine and cosine of an angle and the sine and cosine graphs. (F-IF.7e, F-TF.8)

4. **Students will use the definition of a radian to construct an angle that measures one radian.  They will also determine the number of radians in a full circle.** (F-TF.1)

5. **Students will use their knowledge of special triangles and reference angles to label the exact values of the coordinates on a unit circle.** (F-TF.2, F-TF.3)

6. Students will recognize the connection between the tangent of an angle in the unit circle and the slope of the corresponding radius and they will use a unit circle to generate a graph of t (θ) = tanθ. (F-IF.7e)

7. Students will apply their understanding of transforming parent graphs to the sine and cosine functions and they will generate general equations for the family of sine and cosine functions of the form y = asin(x − h) + k. (F-IF.7e, F-IF.9, F-BF.3, F-TF.5)

8. Students will determine the placement of the parameter b in the general equation for sine and cosine and they will also identify the period of cyclic situations. (F-IF.7e, F-IF.9, F-BF.3, F-TF.5)

9. Students will find equations for transformed sine curves and will graph transformed sine functions. (F-IF.7e, F-IF.9, F-BF.3, F-TF.5)

10. Students will consolidate their understanding of the connections between cyclic graphs and their equations and they will determine that sine and cosine functions are just horizontally shifted versions of each other. (F-IF.7e, F-IF.9, F-BF.3, F-TF.5)

**Unit 3: Matrices** **(approximately 18 days)**

**Description:** In this unit, students will be introduced to matrices. They will learn about matrix arithmetic and learn how matrices can be used in real-world applications problems. They will also use matrices to solve systems of equations and use the matrix features on their calculator.

**Standards**

1. Students add, subtract, and do scalar multiplication with matrices. (N-VM.7 (+), N-VM.8 (+), N-VM.10 (+))

2. Students do matrix multiplication. (N-VM.8 (+), N-VM.9 (+), N-VM.10 (+))

3. Students will use matrix operations to solve application problems. (N-VM.6 (+))

4. Students will use their graphing calculator to perform operations with matrices. (N-VM.7 (+), N-VM.8 (+), N-VM.9 (+), N-VM.10 (+))

5. Students will find the identity element for a matrix and consider inverses for matrices. (N-VM.10 (+), A-REI.9 (+))

6. Students will write systems of equations as matrix equations. (A-REI.8 (+))

7. **Students will solve systems of equations using matrices and the matrix features on their graphing calculators.** (A-REI.9 (+))

**Unit 4: Analytic Trigonometry** **(approximately 28 days)**

**Description:** In this unit, students will extend their previous work with the sine, cosine and tangent functions to define the reciprocal trigonometric functions and they will derive and use the Pythagorean identities. Then they will use these ideas to simplify trigonometric expressions and verify trigonometric identies.

**Standards**

1. Students will develop the Fundamental Pythagorean Identity. (F-TF.8)

2. Students will find the domain, range, and intercepts for the sine and cosine functions. (F-TF.2)

3. Students will be introduced to the reciprocal trig functions.

4. Students will find values for these reciprocal trig functions using the unit circle and a calculator.

5. **Students will simplify trig expressions by rewriting the functions in terms of sine and cosine.**

6. Students will solve trig equations whose solutions are special angles on the unit circle. (F-TF.3 (+))

7. Students will learn about angular frequency and its relationship to period.

8. Students will derive the other two Pythagorean identities.

9. Students will verify trig identies.

10. Students will model real-world situations using sinusoidal functions. (F-TF.5)

11. Students will investigate graphs of new functions that are made by combining trig functions by addition and multiplication.

**Unit 5: Finding the Area Under a Curve** **(approximately 24 days)**

**Description:** The main focus of this unit is finding the area under a curve. This concept is one of the major themes of calculus. The goal is to understand what the area under a curve represents and how to approximate it using rectangles and trapezoids. Along the way, students will further their understanding of piecewise functions, summation, and programming on the graphing calculator.

**Standards**

1. Students will work with piecewise functions and learn to graph them on their calculator.

2. Students will develop an informal definition of continuity.

3. Students will investigate the shifting of piecewise and periodic functions.

4. Students will calculate sums by expanding sigma notation.

5. Students will write a program on the graphing calculator to calculate sums.

6. Students will estimate the area under a curve and develop notation for describing that area.

7. Students will discover that the area under a velocity curve represents distance.

8. **Students will calculate estimates of the area under a curve using right-hand and left-hand rectangles.**

9. Students will further their understanding of what area under a curve may represent and understand that area below the *x*-axis is negative.

10. Students will investigate how vertical and horizontal shifts affect the area under a curve.

11. Students will calculate the area under a curve using trapezoids and compare this result to the results obtained using right- and left-hand rectangles.

12. Students will investigate area under a curve using midpoint rectangles.

13. Students will apply area models to real-world situations.

**Unit 6: Introduction to Limits** **(approximately 18 days)**

**Description:** In this unit, students solve problems involving direct and inverse variation. They will investigate rational functions learn how to rewrite them in more useful forms. They will work with reciprocals of functions and use what they learn to graph secant and cosecant.  They will be introduced to the concept of a limit and they will look at limits from several perspectives including geometry, graphs, tables, and algebra. They will use these ideas to develop the formal definition of continuity.

**Standards**

1. Students will formally define and apply inverse and direct variation.

2. Students will rewrite rational functions into shifts of reciprocal functions.

3. Students will explore how the graph of a function is can be used to graph its reciprocal function.

4. **Students will develop an intuitive idea of a limit.**

5. Students will interpret limit statements written with proper notation.

6. Students will explore limits using tables.

7. Students will understand the necessary conditions for a limit to exist and find the limits when they do exist.

8. Students will see how limits lead to a formal definition of continuity.

**Unit 7: Extending Periodic Functions** **(approximately 24 days)**

**Description:** In this unit, students will solve trigonometric equations by looking at a graph, the unit circle, and using the inverse functions on their calculator.  They will see how several trigonometric equations and applications have more than one solution and they will investigate the inverse trigonometric functions. They will be extending their modeling of periodic functions to more complex situations and they will work with other trigonometric formulas and identities to solve more complex trigonometric equations.

**Standards**

1. Students will understand that the number of solutions for trig equations depends on the domain of the problem. (F-TF.2)

2. Students will solve trig equations and develop the notation for expressing all solutions. (F-TF.2)

3. Students will graph the relations for inverse sine and cosine. (F-TF.6 (+))

4. Students will restrict the range for inverse sine and inverse cosine to make them functions. (F-TF.6 (+))

5. Students will use inverse functions to solve for angles which are not special angles on the unit circle. (F-TF.7 (+))

6. Students will investigate the ambiguous case for the Law of Sines. (G-SRT.11 (+))

7. Students will graph tangent and its inverse. (F-TF.6 (+))

8. Students will use tangent to find the angle on inclination for linear equations.

9. **Students will develop trigonometric models for real-world applications.**

10. Students will develop and use the angle sum and difference formulas to simplify expressions. (F-TF.9 (+))

11. Students will use the angle sum and difference formulas to develop the formulas for double-angle formulas. (F-TF.9 (+))

12. Students will use the double-angle formulas to solve trig equations.

13. Students will use the formula for cos (2*α*) to develop the half-angle formula.

14. Students will solve trig equations using identities and algebraic simplifications. (F-TF.9 (+))

**Unit 8: Advanced Algebra Techniques for College Mathematics** **(approx. 12 days)**

**Description:** In this unit, students will learn several problem-solving techniques including techniques for simplifying expressions, purposeful substitution, and polynomial division. They will also be introduced to Pascal’s Triangle and see how it can be used as a problem solving tool. They will work with finite arithmetic and geometric series and apply them to real world problems.

**Standards**

1. Students will use substitution to simplify algebraic expressions.

2. **Students will learn to divide polynomials both from algebraic and geometric viewpoints.** (A-APR.6)

3. Students will use Pascal’s Triangle and the binomial formula to expand polynomials. (A-APR.5 (+))

4. Students will develop and use a formula for the sum of an arithmetic series. (A-SSE.2)

5. Students will develop and use formula for the sum of a geometric series. (A-SSE.4)